Characteristic Impedance of Rectangular Coaxial Transmission Lines

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Summary—**The characteristic impedance of rectangular coaxial transmission lines can be readily and accurately computed by a simple equation when the capacitance per unit length is known. While the capacitance per unit length of the parallel sides is easily calculated, the calculation of the corner capacitance is a more difficult problem. This problem has been solved by Skiles and Higgins, using orthonormal block analysis. By using their formulas, the corner capacitances for a wide range of all variables were evaluated by means of a digital computer. The results are catalogued in graphical form.**

I. Introduction

RECTANGULAR coaxial transmission lines are becoming of increasing importance as high-quality transitions from round coaxial transmission lines to strip transmission lines [1], as desirable structures in which to mount garnet materials for solid-state microwave devices [2], and as low-capacitance mounts for varactor diodes [3].

The analysis of such lines has been the subject of substantial research in attempts to find a simple and precise expression for the characteristic impedance of high-current busses in the rectangular coaxial configuration. This work is referenced by Skiles and Higgins [4] in the same paper in which they give the technique for calculating the characteristic impedance for all symmetrical configurations to any desired degree of precision. This technique is known as orthonormal block analysis, and will be discussed in Section II. The computing and cataloging of a large number of cases so that the desired information may be obtained directly remained undone.

A method of cataloging was derived from the analytic solutions of Chen [5]. The diversity of Chen's solutions and the conditions required for their accuracy made it difficult to establish which solution best described each problem. In the attempt to unify these solutions, a cataloging system [6] was devised in the form of a single graph. Chen's analytic solutions were exact for all cases of a rectangular coaxial transmission line in which $w/h>1$ and $b/g>1$ [dimensional symbols as in Fig. 1(a)]. Cohn derived an exact solution for the case in which $b/g=0$ [7] and a very accurate solution for the case in which $g/h<0.2$ [8]. Anderson [9] derived an exact expression for the case in which $g/h=1.0$. The solutions of Getsinger [10] are accurate for most cases in which $g/h\leq 1.5$ and for some cases in which $g/h>1.5$ ($b/g>0.8$). For some values of $g/h>1.5$ and at $w/h>1$, no accurate expressions exist for the characteristic impedance of rectangular coaxial transmission lines. In this paper, solutions are given with a known degree of accuracy for a wide range of all variables.

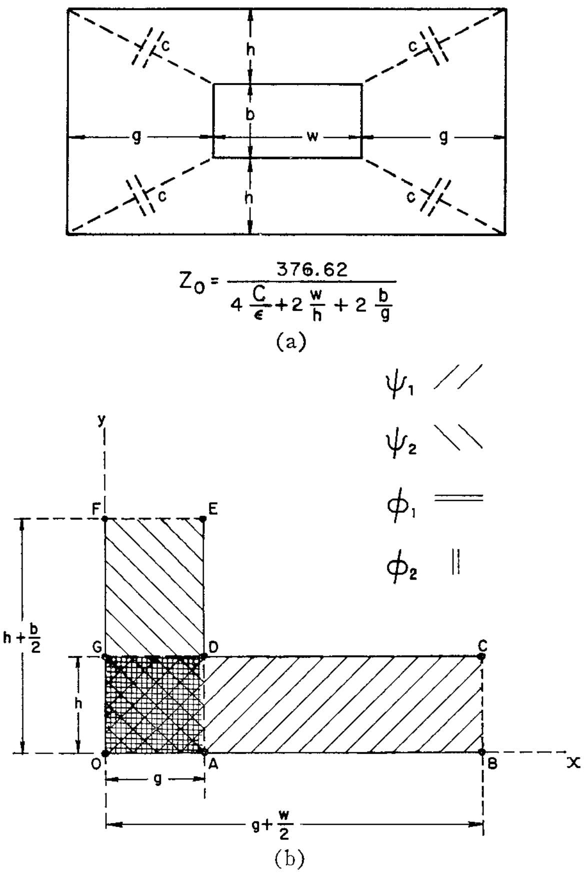

Fig. 1—(a) Cross-sectional view of rectangular coaxial transmission line and the equation determining its characteristic impedance with an air dielectric. The "corner capacitance C" is given in Figs. 6–11. (b) Quarter section of rectangular coaxial transmission line showing partial potentials in each region.

The characteristic impedance Z_0 of a TEM transmission line is a function of its capacitance per unit length C', according to the relationship $Z_0=(cC')^{-1}$ in which c is the speed of light in the dielectric between the conductors. In a rectangular coaxial transmission line, a substantial part of the capacitance per unit length is attributable to the capacitance between parallel planes. The remainder is contributed by the fringing capaci-

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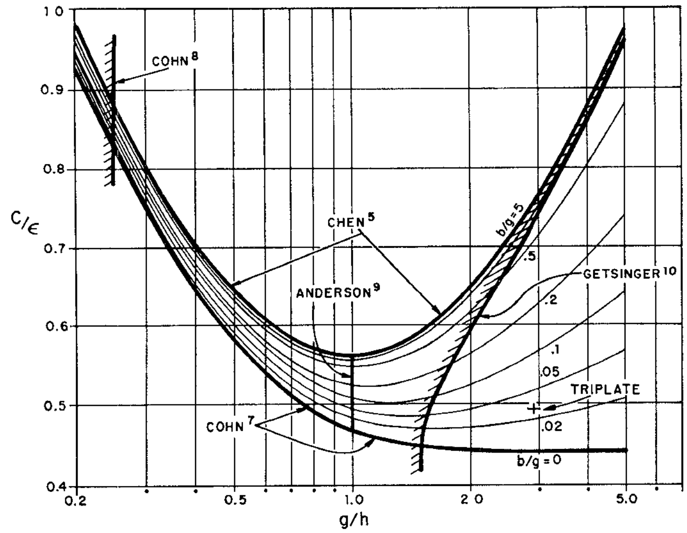

Fig. 2—Corner capacitance for $w/h=5$ showing regions of applicability for other theoretical derivations. The ratios for 50 ohm Triplate stripline is also shown.

tance at the corners. This corner capacitance is the only difficult factor to determine. Fig. 2 is a graph of this corner capacitance in which $w/h \geq 1$ and also illustrates the regions over which the above mentioned analyses pertain.

II. Method of Orthonormal Block Analysis

To obtain the capacitance C' and consequently the corner capacitance C, Skiles and Higgins [4], and Skiles [11] have used a mathematical method known as orthonormal block analysis [12], [13]. In this method, the region to be analyzed is considered to be composed of smaller regions (blocks) that overlap each other. For each block, a primary potential function satisfying the appropriate partial differential equation is assumed to be expressible in orthonormal functions. Similarly, two auxiliary potential functions are thus expressed. These auxiliary functions satisfy some of the same boundary conditions as the primary functions.

By making the appropriate finite Fourier transforms of the partial differential equations, one obtains relationships involving the unknown coefficients of the potential expansions. From the use of boundary conditions and of elimination, these relationships can be reduced to dependence upon two infinite systems of simultaneous linear equations involving only two sets of unknown quantities. From the solutions of these systems, values of all the coefficients are obtained, and all the potentials are thereby determined.

For the specific case of the capacitance C' of a rectangular coaxial transmission line, these potentials are substituted into a general integral for capacitance, and a subsequent integration is made around the contour of the region. By way of illustrating the method as applied by Skiles [11] to this case, an outline is presented in Section III.

III. The Capacitance of a Rectangular Coaxial Transmission Line

Laplace's Equation and Potential Functions

For the rectangular coaxial transmission line, the region to be considered is that between the inner and outer conductors, as shown in Fig. 1(a). However, from the symmetry inherent in the line cross section, it is sufficient to consider only one quarter of it, as illustrated in Fig. 1(b). In this region, a potential function U is assumed to exist, which has a zero value on the outer conductor and a value of U_0 on the inner conductor. Since the field contains no sources, the U satisfies Laplace's equation:

$$\nabla^2 U = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = 0. \tag{1}$$

The primary and auxiliary potentials, which, taken together or separately as required, comprise the potential U, may be expressed in the orthonormal series;

$$\left.\begin{aligned}
\Psi_1(x, y) &= \sum_{k=1}^{\infty} f_k(x) \sin (k\pi y/h), \quad \text{Region } OBCG \\
\Psi_2(x, y) &= \sum_{k=1}^{\infty} v_k(y)\, sin\, (k\pi x/g), \quad \text{Region } OAEF \\
\Phi_1(x, y) &= \sum_{k=1}^{\infty} \phi_k(x) \sin (k\pi y/h), \quad \text{Region } OADG \\
\Phi_2(x, y) &= \sum_{k=1}^{\infty} w_k(y) \sin (k\pi x/g), \quad \text{Region } OADG
\end{aligned}\right\}, \tag{2}$$

where $f_k(x)$, $v_k(y)$, $\phi_k(y)$, and $w_k(y)$ are unknown coefficients to be determined by the boundary conditions. It will be assumed that these partial potentials Ψ_1, Ψ_2, Φ_1, and Φ_2 also satisfy Laplace's equation. In terms of the partial potentials, the potential U becomes

$$
\left.
\begin{aligned}
U &= U_1(x, y) = \Psi_1(x, y), && \text{Region } ABCD \\
&&& g < x < g + w/2 \\
&&& o < y < h \\
U &= U_1(x, y) = \Psi_1(x, y) + \Phi_1(x, y), && \text{Region } OADG \\
&&& o < x < g \\
&&& o < y < h \\
U &= U_2(x, y) = \Psi_2(x, y), && \text{Region } DEFG \\
&&& o < x < g \\
&&& h < y < h + b/2 \\
U &= U_2(x, y) = \Psi_2(x, y) + \Phi_2(x, y), && \text{Region } OADG \\
&&& o < x < g \\
&&& o < y < h
\end{aligned}
\right\} . \tag{3}
$$

Boundary Conditions

To determine the unknown coefficients of the partial potentials (2), the boundary conditions are required. Thus

1) At $y=0$, we have

$$1°\text{–}1.\ U_1(x, o) = \Psi_1(x, o) + \Phi_1(x, o) = 0, \quad o < x < g + w/2. \tag{4}$$

If we take

$$1°\text{–}2.\ \Psi_1(x, o) = 0, \qquad o < x < g + w/2,$$

then we have

$$1°\text{–}3.\ \Phi_1(x, 0) = 0, \qquad o < x < g.$$

Furthermore, we have

$$1°\text{–}4.\ U_2(x, o) = \Psi_2(x, o) + \Phi_2(x, o) = 0, \quad o < x < g.$$

2) At $y=h$, taking

$$2°\text{–}1.\ \Psi_1(x, h) = U_0, \qquad o < x < g + w/2$$

we have

$$2°\text{–}2.\ U_1(x, h) = U_0 + \Phi_1(x, h), \qquad o < x < g.$$

Since $\Psi_2(x, y)$ is continuous across $y=h$, then from the last two equations of (3), it follows that

$$2°\text{–}3.\ \Phi_2(x, h) = 0, \qquad o < x < g.$$

The relation between the functions U_1 and U_2 at $y=h$ is

$$2°\text{–}4.\ U_1(x, h) = U_2(x, h), \qquad o < x < g.$$

Since Ψ_2 and $\partial\Psi_2/\partial y$ are continuous across $y=h$, then, for no discontinuity in $\partial U_2/\partial y$, it follows from the last two equations of (3), that

$$2°\text{–}5.\ \left(\frac{\partial \Phi_2}{\partial y}\right)_{y=h} = 0, \qquad o < x < g.$$

3) At $y=h+b/2$, for no discontinuity in $\partial U/\partial y$, we must have

$$3°\text{–}1.\ \left(\frac{\partial \Psi_2}{\partial y}\right)_{y=b/2} = 0, \qquad o < x < g.$$

The boundary conditions at $x=o$, $x=g$, and $x=g+w/2$ can be similarly obtained.

Coefficients of Potential Expansions

By making finite Fourier transforms of Laplace's partial differential equation,

$$\frac{2}{d}\int_0^d \nabla^2 U' \sin\left(\frac{k\pi z}{d}\right) dz = 0, \tag{5}$$

where d is g or h, z is x or y as the case may be, and U' is any of the partial potentials listed in (2), we obtain the following relationships involving the unknown coefficients of the potential expansions:

$$
\left.
\begin{aligned}
f_k(x) &= A_k \sinh\left(\frac{k\pi x}{h}\right) + B_k' \cosh\left(\frac{k\pi x}{h}\right) - \frac{2}{k\pi}(-1)^k U_0 \\
v_k(y) &= M_k \sinh\left(\frac{k\pi y}{g}\right) + N_k \cosh\left(\frac{k\pi y}{g}\right) - \frac{2}{k\pi}(-1)^k U_0 \\
\phi_k(x) &= D_k' \sinh\left(\frac{k\pi x}{h}\right) + C_k \cosh\left(\frac{k\pi x}{h}\right) + (-1)^k \frac{2U_0}{k\pi} + (-1)^k \frac{2}{\pi}\sum_{p=1}^{\infty} v_p(h)\left[\left(p\frac{h}{g}\right)^2 + k^2\right]^{-1} \\
&\quad \cdot \left[\left(p\frac{h}{g}\right)\sinh\left(\frac{k\pi x}{h}\right) - k\sin\left(\frac{p\pi x}{g}\right)\right] \\
w_k(y) &= L_k \sinh\left(\frac{k\pi y}{g}\right) + F_k \cosh\left(\frac{k\pi y}{g}\right) + (-1)^k \frac{2U_0}{k\pi} + (-1)^k \frac{2}{\pi}\sum_{p=1}^{\infty} f_p(g)\left[\left(p\frac{g}{h}\right)^2 + k^2\right]^{-1} \\
&\quad \cdot \left[\left(p\frac{g}{h}\right)^2 \sinh\left(\frac{k\pi y}{g}\right) - k\sin\left(\frac{p\pi y}{h}\right)\right]
\end{aligned}
\right\} . \tag{6}
$$

Upon applying certain of the boundary conditions, eliminating coefficients, and after several transformations, one obtains two sets of infinite systems of simultaneous equations,

$$\left.\begin{aligned} X_k &= \sum_{p=1}^{\infty} a_{kp} Y_p + B_k \\ Y_k &= \sum_{p=1}^{\infty} c_{kp} X_p + D_k \end{aligned}\right\}, \tag{7}$$

from which the coefficients can be obtained. In (7),

$$\left.\begin{aligned} X_k &= k \frac{h(-1)^k B_k'}{g \sinh\left(\frac{kg}{h}\right)} \\ Y_k &= k \frac{h(-1)^k N_k}{g \sinh\left(\frac{k\pi h}{g}\right)} \\ a_{kp} &= \frac{2k}{\pi} \frac{h}{g} \frac{m_p}{\left(\frac{h}{g}\right)^2 p^2 + k^2}; \quad B_k = \frac{2}{k\pi^2}\left(\frac{h}{g}\right)^2 \\ c_{kp} &= \frac{2k}{\pi} \frac{g}{h} \frac{n_p}{\left(\frac{g}{h}\right)^2 p^2 + k^2}; \quad D_k = \frac{2}{k\pi^2} \\ m_p &= \sinh\left(\frac{p\pi h}{g}\right) \cosh\left(\frac{p\pi b}{2g}\right) \Big/ \cosh\left[p\pi\left(\frac{b}{2g}\right) + \frac{h}{g}\right] \\ n_p &= \sinh\left(\frac{p\pi g}{h}\right) \cosh\left(\frac{p\pi g}{2h}\right) \Big/ \cosh\left[p\pi\left(\frac{w}{2h} + \frac{g}{h}\right)\right] \end{aligned}\right\}. \tag{8}$$

Capacitance of Rectangular Coaxial Transmission Line

Since from (6) and the boundary conditions (4) it can be shown that $f_k(x)$ and $v_k(y)$ depend upon B_k' and N_k, respectively, then,

$$\left.\begin{aligned} \Psi_1 &= U_0 \frac{y}{h} + \sum_{k=1}^{\infty} (-1)^k U_0 \frac{g}{h} \frac{Y_k}{k} \sin\left(\frac{k\pi y}{h}\right) \sinh\left(\frac{k\pi g}{h}\right) \operatorname{sech}\left[k\pi\left(\frac{w}{2h} + \frac{g}{h}\right)\right] \cosh\left[k\pi \frac{(x - g - w/2)}{h}\right] \\ \Psi_2 &= U_0 \frac{x}{g} + \sum_{k=1}^{\infty} (-1)^k U_0 \frac{g}{h} \frac{Y_k}{k} \sin\left(\frac{k\pi x}{g}\right) \sinh\left(k\pi \frac{h}{g}\right) \operatorname{sech}\left[k\pi\left(\frac{b}{2g} + \frac{h}{g}\right)\right] \cosh\left[k\pi \frac{(h - h - b/2)}{g}\right] \end{aligned}\right\}. \tag{9}$$

From a consideration of the region involved, it can be shown that these two partial potentials are the only ones that need to be used in obtaining the capacitance of the line.

Now the capacitance per unit length of a line, in rationalized mks units, is given by

$$C' = \frac{\epsilon}{U_0}\int_S \left(\frac{\partial U}{\partial n}\right)_S dS, \tag{10}$$

where $\partial U/\partial n$ is the normal derivative of the potential and S is the length of the perimeter of the line cross section. With the values of U as given by (9),

$$C' = 4\epsilon\left\{\frac{w}{2h} + \frac{b}{2g} + \frac{1}{3}\left[\frac{h}{g} + \frac{g}{h}\right] - \frac{1}{\pi}\left[\left(\frac{g}{h}\right)^2 \sum_{p=1}^{\infty} \frac{Y_p}{p^2} m_p + \sum_{p=1}^{\infty} \frac{X_p}{p^2} n_p\right]\right\}, \tag{11}$$

where m_p and n_p are given in (8).

IV. The Corner Capacitance

The quantity $4\epsilon(w/2h+b/2g)$ is that which is normally associated with the capacitance of parallel plates. Thus the remainder of the right term of (11) may be considered as the capacitance associated with the four corners. For one corner, the capacitance is

$$C = \epsilon\left\{\frac{1}{3}\left(\frac{h}{g} + \frac{g}{h}\right) - \frac{1}{\pi}\left[\left(\frac{g}{h}\right)^2 \sum_{p=1}^{\infty} \frac{y_p}{p^2} m_p + \sum_{p=1}^{\infty} \frac{X_p}{p^2} n_p\right]\right\}. \tag{12}$$

In evaluating C, the two infinite systems of simultaneous equations (7) are solved by using finite systems derived from these equations. Upper and lower bounds of the X_k's and Y_k's are found, and are then improved by iteration techniques. The method used here is partly outlined by Skiles [11]; the theory is given by Koyalovich [14], and Kantorovich and Krylov [15]. To obtain the first approximation of the lower bounds, we write (7) in the following way:

$$\left.\begin{aligned} \tilde{X}_k &= \sum_{p=1}^{N} a_{kp}\tilde{Y}_p + B_k \\ \tilde{Y}_k &= \sum_{p=1}^{N} c_{kp}\tilde{X}_p + D_k \end{aligned}\right\}, \quad k = 1, 2, \cdots N \tag{13}$$

and

$$\left.\begin{aligned} \tilde{X}_k &= 0 \\ \tilde{Y}_k &= 0 \end{aligned}\right\}, \quad k = N+1, N+2, \cdots.$$

If the second of (13) is substituted into the first, then,

$$\tilde{X}_k = \sum_{q=1}^{N}\left(\sum_{p=1}^{N} a_{kp}c_{pq}\right)\tilde{X}_q + \sum_{p=1}^{N} a_{kp}D_p + B_k, \quad k = 1, 2, \cdots N \tag{14}$$

is obtained. The coefficients and free terms in (14) can be readily evaluated. The $\tilde{Y}_k$ are obtained by substituting the values of the $\tilde{X}_k$'s into the second of (13).

To obtain the first approximation of the upper bounds, we substitute K for all the X's and Y's in (7) and solve for the largest value of K. Thus, we obobtain,

$$K \geq B_k \Big/ \left(1 - \sum_{p=1}^{\infty} a_{kp}\right) \tag{15}$$

and

$$K \leq D_k \Big/ \left(1 - \sum_{p=1}^{\infty} c_{kp}\right)$$

and use the largest K value. According to theory,

$$\left.\begin{aligned} X_k &< K \\ Y_k &< K \end{aligned}\right\}, \quad k = 1, 2, \cdots. \tag{16}$$

More exact upper bounds on the X_k's and Y_k's are found by using (7) in which K replaces the X_y's and Y_k's for $k>N$. Thus, we have the equations

$$\left.\begin{aligned} \overline{X}_k &= \sum_{p=1}^{N} a_{kp}\overline{Y}_p + K\sum_{p=N+1}^{\infty} a_{kp} + B_k \\ \overline{Y}_k &= \sum_{p=1}^{N} c_{kp}\overline{X}_p + K\sum_{p=N+1}^{\infty} c_{kp} + D_k \end{aligned}\right\}, k = 1, 2, \cdots N, \tag{17}$$

with

$$\left.\begin{aligned} \overline{X}_k &= K \\ \overline{Y}_k &= K \end{aligned}\right\}, \quad k = N+1, N+2, \cdots.$$

To solve these equations, we substitute the value of Y_k from the second of equations (17) into the first, obtaining

$$\overline{X}_k = \sum_{q=1}^{N}\left(\sum_{p=1}^{N} a_{kp}c_{pq}\right)\overline{X}_p + K\sum_{p=N+1}^{\infty}\left(a_{kp} + \sum_{q=1}^{N} c_{qp}\,a_{kq}\right) + \sum_{p=1}^{N} a_{kp}D_p + B_k. \tag{18}$$

It can be shown from theory[1] that the lower bounds on the two infinite systems of simultaneous equations (7) cannot be improved by an iteration process, but an improvement can be realized in the case of the upper bounds. The equations used for improving the upper bounds are

$$\left.\begin{aligned} \overline{X}_k^{i+1} &= \sum_{p=1}^{N} a_{kp}\overline{Y}_p^{i} + H^{i+1}\sum_{p=N+1}^{\infty} a_{kp} + B_k \\ \overline{Y}_k^{i+1} &= \sum_{p=1}^{N} c_{kp}\overline{X}_p^{i} + H^{i+1}\sum_{p=N+1}^{\infty} c_{kp} + D_k \end{aligned}\right\}, \quad k = 1, 2, \cdots N \tag{19}$$

and

$$\left.\begin{aligned} \overline{X}_k^{i+1} &= H^{i+1} \\ \overline{Y}_k^{i+1} &= H^{i+1} \end{aligned}\right\}, \quad k = N+1, N+2, \cdots$$

[1] See Skiles [11], p. 112.

where H^{i+1} is taken to be the largest of W_k^{i+1} or V_k^{i+1} for all $k > N$, and

$$V_k^{i+1} = \left(B_k + \sum_{p=1}^{N} a_{kp}\overline{Y}_p^{i}\right) \Big/ \left(1 - \sum_{p=N+1}^{\infty} a_{kp}\right)$$

$$W_k^{i+1} = \left(D_k + \sum_{p=1}^{N} c_{kp}\overline{X}_p^{i}\right) \Big/ \left(1 - \sum_{p=N+1}^{\infty} c_{kp}\right).$$

In the above equations (i) indicates the ith approximation of the upper bounds.

By means of an electronic digital computer, the value of $\tilde{X}_k$, $\tilde{Y}_k$, and $\bar{X}_k$, $\overline{Y}_k$ can be readily determined for numerous values of the parameters b, w, g, and h. If b and w are zero or close to zero, then practical difficulties are encountered in obtaining the upper bounds of X_k and Y_k. For example, if b is close to zero, then the series

$$\sum_{p=N+1}^{\infty} a_{kp}$$

in (18) is difficult to evaluate, since m_p in the numerator of a_{kp} approaches 0.5 slowly. This prevents the practical application of a summing technique to the remainder of the series when $m_p = 0.5$. On the other hand, if $b = 0$, m_p approaches unity fairly rapidly for large values of p. This permits the use of a summing technique, but the bounds cannot be readily improved. The essential improvement results from using a greater number of terms in the simultaneous equations. To obtain a satisfactory improvement in the bounds, an excessive number of terms is required. Thus, the error associated with the curves for $b/g = 0$ in the various graphs is greater than that associated with the other curves for the corner capacitance.

From (12), it will be observed that an upper bound on the corner capacitance C is obtained for lower bounds on the X_k's and Y_k's, whereas a lower bound is given by upper bounds on the same quantities.

V. Graphs of Corner Capacitance

Because solutions had to be found for a large number of points to obtain the graphs and because computer time increased rapidly when a large number of equations is used for each point, it was desirable to limit the number of equations used. The variation of the average C/ϵ and the number of equations required to obtain a maximum deviation[2] of 0.004 were estimated. Fig. 3 shows that, although the deviation changes considerably with the number of equations, the average C/ϵ does not, indicating that the magnitude of difference between the average C/ϵ and the exact C/ϵ is much less than the deviation. Fig. 4 demonstrates the diminishing deviation with a large number of equations. Because the deviation diminished so slowly with increasing numbers of equations, only fifteen equations were used for solving for the average C/ϵ, which is used in Figs. 5–11.

[2] Maximum deviation is maximum C/ϵ less average C/ϵ.

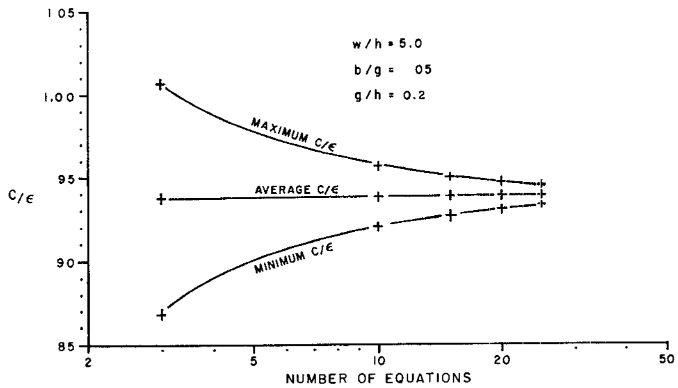

Fig. 3—Maximum, minimum, and average corner capacitance from an increasing number of equations.

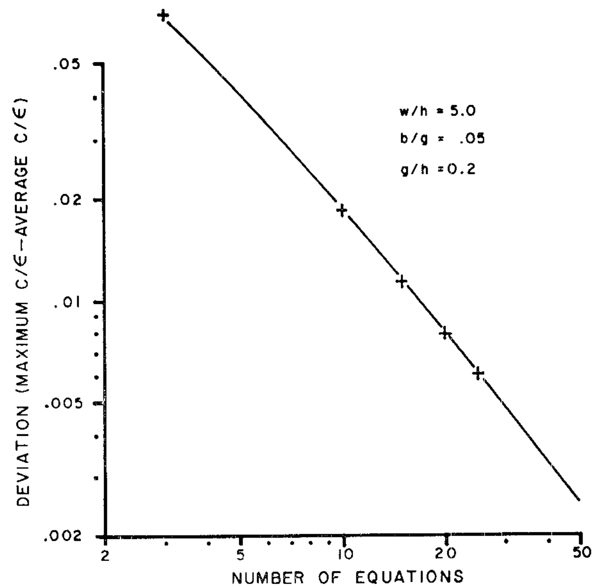

Fig. 4—Decreasing deviation in corner capacitance with increasing number of equations. The deviation is projected to show the number of equations required for a particular accuracy.

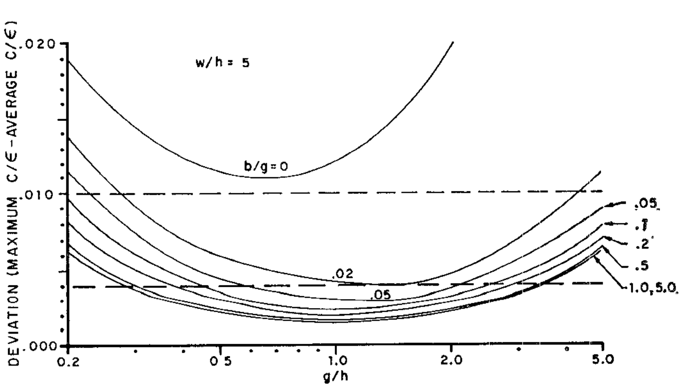

Fig. 5—Deviation for fifteen equations for all curves of Fig. 6.

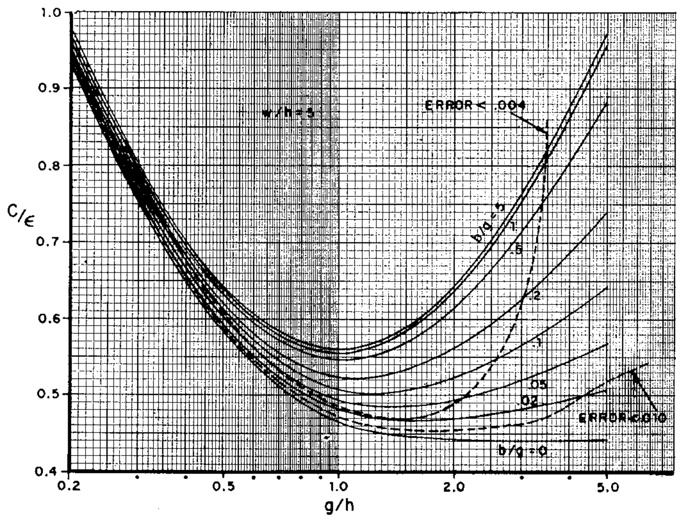

Fig. 6—Corner capacitance for $w/h=5$.

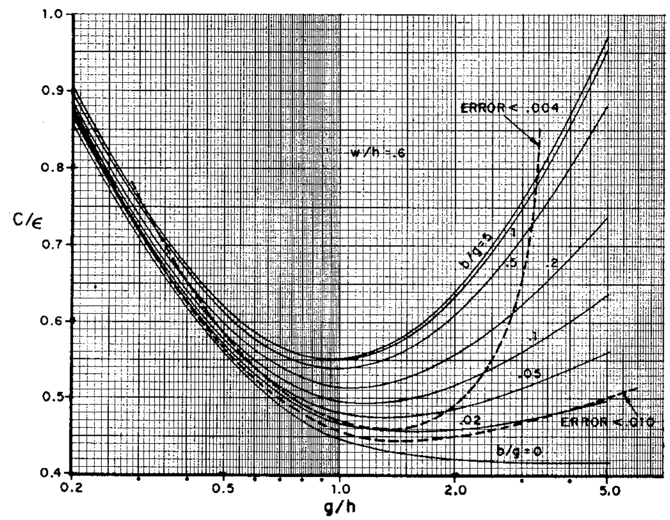

Fig. 9—Corner capacitance for $w/h=0.6$.

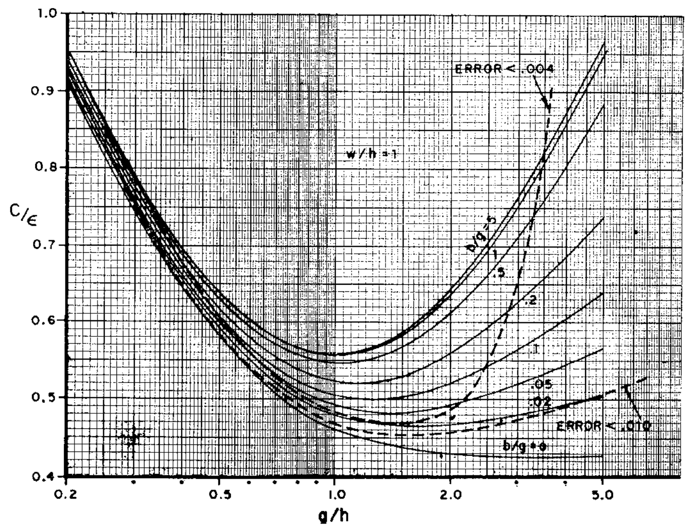

Fig. 7—Corner capacitance for $w/h=1$.

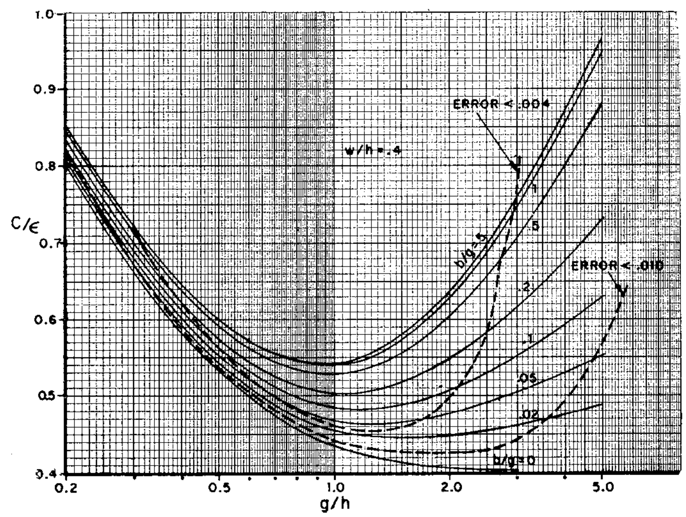

Fig. 10—Corner capacitance for $w/h=0.4$.

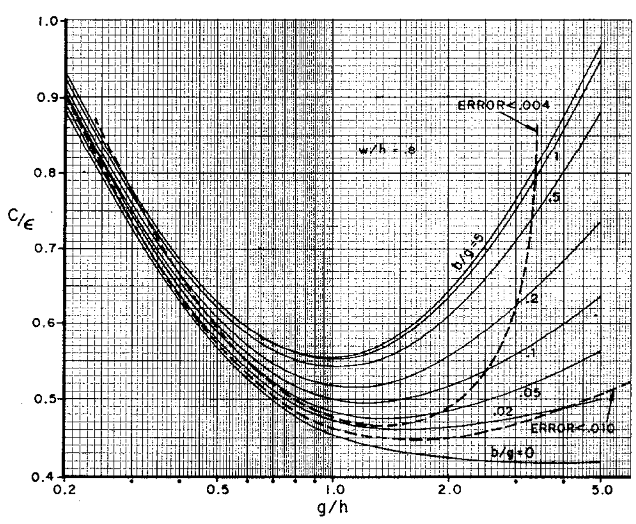

Fig. 8—Corner capacitance for $w/h=0.8$.

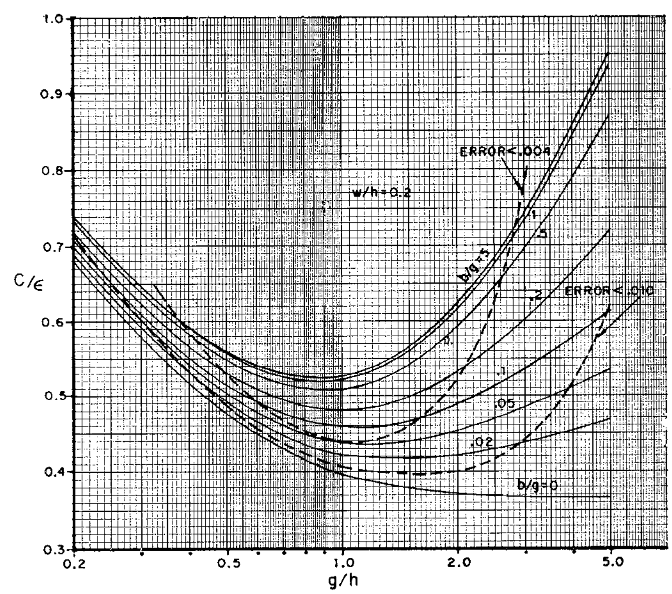

Fig. 11—Corner capacitance for $w/h=0.2$.

Fig. 5 demonstrates the variation of the deviation with fifteen equations over the range of parameters considered in Fig. 6. The deviation limits of 0.004 and 0.010 are shown on Figs. 6–11. These figures with interpolation then give the average corner capacitance for a wide range of parameters with a known degree of accuracy. For example, C/ϵ for $w/h=2.0$, $b/g=0.5$, and $g/h=2.0$ can be obtained by interpolation from values obtained from the graphs. Fig. 6 for $w/h=5$ gives $C/\epsilon=0.614\pm 0.004$. Fig. 7 for $w/h=1$ gives $C/\epsilon=0.612 \pm 0.004$. Interpolation for $w/h=2$ then gives $C/\epsilon=0.613 \pm 0.005$. Substituting these values into the equation of Fig. 1(a)

$$Z_0 = \frac{376.62}{7.452 \pm 0.020} = 50.54 \pm 0.136 \text{ ohms}$$

$$= 50.54 \text{ ohms} \pm 0.27 \text{ per cent.}$$

The error here is less than 0.3 per cent and may be much less. Since the error in characteristic impedance is lower than the error in corner capacitance, the graphs provide greater accuracy in Z_0 than is apparent from the graphs alone.

VI. Comments

The values on the graphs [6] that were obtained using Chen's approximation are reasonably close to the values given by the exact solution of Skiles and Higgins.

The error in computing the curves of $b/g=0$ in the various graphs is greater than that associated with the other curves ($b/g=0.02$, 0.05, etc.).

The characteristic impedance of rectangular coaxial transmission lines can be quickly and accurately determined from the graphs presented here.

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